Crossover to Cluster Plasma in the Gas of Quark-Gluon Bags

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Abstract

We study a smooth crossover transition in the gas of quark-gluon bags. The equation of state at high temperature is that of the quark-gluon plasma. However, the system consists of the bags with finite volumes which are defined by the model parameters of the mass-volume bag spectrum. Possible structures in this *cluster* quark-gluon plasma are classified.

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I. INTRODUCTION

A connection of the bag model [1] with statistical description of strongly interacting matter at high energy density has a long history [2]. The possibility of phase transitions in the gas of quark-gluon bags was demonstrated for the first time in Ref. [3]. Further studies allowed to obtain the 1st, 2nd, and higher order transitions [4, 5, 6, 7, 8]. A possibility of no phase transitions was also pointed out [4, 5]. Recently it was suggested [9] to model a smooth crossover transition by the gas of quark-gluon bags. Inspired by this suggestion we study in more details the high temperature behavior of the system of quark-gluon bags in case of the crossover. Note that the present and future experimental facilities such as RHIC and LHC produce strongly interacting matter in the crossover region of the QCD phase diagram [10]. Moreover, the location of the (tri)critical point that ends the line of the 1st order phase transition is unknown [11]. It can happen that nucleus-nucleus collisions at SPS and FAIR also partially enter the crossover region.

Section II gives a short overview of the gas of bags with excluded volume. In Section III the phase diagram of the model parameters in the mass-volume bag spectrum is considered. The average volume of the bags is calculated in the high temperature limit. In Section IV a short summary of the results is presented.

II. GAS OF QUARK-GLUON BAGS

The partition function for gas of quark-gluon bags is the following [3]:

$$Z(V,T) ~=~ \sum_{N=0}^{\infty}\frac{1}{N!}\,\prod_{i=1}^{N}\int dm_i dv_i~\rho(m_i,v_i)~\phi(T,m_i)\left(V-\sum_{j=1}^{N}v_j\right)^{N}\theta\left(V-\sum_{j=1}^{N}v_j\right)~, \quad (1)$$

where V and T are the system volume and temperature respectively, N is number of bags, m_i, v_i are mass and proper volume of i-th bag, $\rho(m_i,v_i)$ is the mass-volume spectrum of bags that will be specified later. The $\phi(T,m_i)$ in Eq. (1) equals:

$$\phi(T,m_i)~\equiv~\frac{1}{2\pi^2}\int_0^{\infty}k^2dk~\exp\left[-~\frac{(k^2~+~m_i^2)^{1/2}}{T}\right]~=~\frac{m_i^2T}{2\pi^2}~K_2\left(\frac{m_i}{T}\right)~, \quad (2)$$

and it has the physical meaning of the particle number density in Boltzmann ideal gas, K_2 in Eq. (2) is the modified Bessel function. The baryonic number and other conserved charges are assumed to be equal to zero. The equation of state can be most easily studied with the help of

the Laplace transform:

$$\hat{Z}(T,s) \;\equiv \int_0^{\infty} dV \exp(-sV)\; Z(V,T) \;=\; [\,s \;-\; f(T,s)]^{-1}\;, \tag{3}$$

where

$$f(T,s) \;=\; \int_0^{\infty} dmdv\; \exp(-vs)\; \rho(m,v)\, \phi(T,m)\;. \tag{4}$$

In the thermodynamic limit, $V \to \infty$, the partition function behaves as $Z(V,T) \cong \exp\left[p\, V/T\right]$, where $p(T)$ is the system pressure. An exponential increase of $Z(V,T)$ on V generates the singularity s^* of the function $\hat{Z}(T,s)$ in variable s. Consequently, one can calculate the pressure knowing only the position of the farthest-right singularity s^*:

$$p(T) \;=\; T \lim_{V\to\infty} \frac{\ln Z(V,T)}{V} \;=\; T\; s^*(T)\;. \tag{5}$$

There is a pole singularity $s^* = s_H$ of $\hat{Z}(T,s)$ calculated from the transcendental equation:

$$s_H(T) \;=\; f(T, s_H(T))\;. \tag{6}$$

Another singular point of $\hat{Z}(T,s)$ denoted as $s_Q(T)$ emerges due to a singularity of the function $f(T,s)$ itself. The system pressure takes then the form [3]:

$$p(T) \;=\; Ts^*(T) \;=\; T\cdot max\{s_H(T), s_Q(T)\}\;, \tag{7}$$

i.e. the farthest-right singularity $s^*(T)$ of $\hat{Z}(T,s)$ (3) can be either the pole singularity $s_H(T)$ (6) or the $s_Q(T)$ singularity of the function $f(T,s)$ (4) itself. The mathematical mechanism for possible phase transition (PT) in the gas of quark-gluon bags is the "collision" of the two singularities, i.e. $s_H(T) = s_Q(T)$ at the PT temperature $T = T_C$.

The crucial ingredient of the model is the form of the mass-volume spectrum $\rho(m,v)$. In the case of a bag filled with the non-interacting massless quarks and gluons[1] one finds [3, 4, 5]:

$$\rho(m,v) \;\simeq\; C\; v^{\gamma}(m-Bv)^{\delta}\; \exp\left[\frac{4}{3}\,\sigma_Q^{1/4}\; v^{1/4}\;(m-Bv)^{3/4}\right]\;, \tag{8}$$

where C, γ, δ and B, the so-called bag constants, are the model parameters, and $\sigma_Q = 95\pi^2/60$ is the Stefan-Boltzmann constant counting gluons (spin, color) and (anti-)quarks (spin, color

[1] This picture is reasonable in the region where both the mass of the bag, m, and the volume of the bag, v, are large. A general form of the mass-volume spectrum function should contain also the low-lying hadron states. However, the type of the PT or the high temperature behavior when the PT is absent are not sensitive to presence of these hadron-like states.

and u, d, s-flavor) degrees of freedom inside the bag. This is the asymptotic expression assumed to be valid for a sufficiently large volume and mass of a bag, $v > V_0$ and $m > Bv + M_0$. The validity limits can be estimated to be $V_0 \approx 1$ fm^3 and $M_0 \approx 2$ GeV [5].

The integral over mass in Eq. (4) can be calculated by the steepest descent estimate and one finds:

$$\begin{aligned} f(T,s) &\simeq C \int_{V_0}^{\infty} dv\ v^{\gamma} \exp\left[-v\left(s - s_Q\right)\right] \int_{M_0}^{\infty} dm\,(m - Bv)^{\delta} \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-\frac{(m-\overline{m})^2}{8\sigma_Q v T^5}\right] \\ &\simeq u(T) \int_{V_0}^{\infty} dv\ v^{2+\gamma+\delta}\ \exp\left[-v\left(s\ - s_Q(T)\right)\right]\ , \end{aligned} \tag{9}$$

where $\overline{m} = v(\sigma_Q T^4 + B)$, $u(T) = C\pi^{-1}\sigma_Q^{\delta+1/2}\ T^{4+4\delta}\ \left(\sigma_Q T^4 + B\right)^{3/2}$ and

$$s_Q(T)\ \equiv\ \frac{1}{3}\,\sigma_Q\,T^3\ -\ \frac{B}{T}\ . \tag{10}$$

III. THE $\gamma - \delta$ PHASE DIAGRAM

The model parameters γ and δ define the presence, location and order of the PT in the gas of quark-gluon bags. This is illustrated by the $\gamma - \delta$ phase diagram in Fig. 1 (*left*).

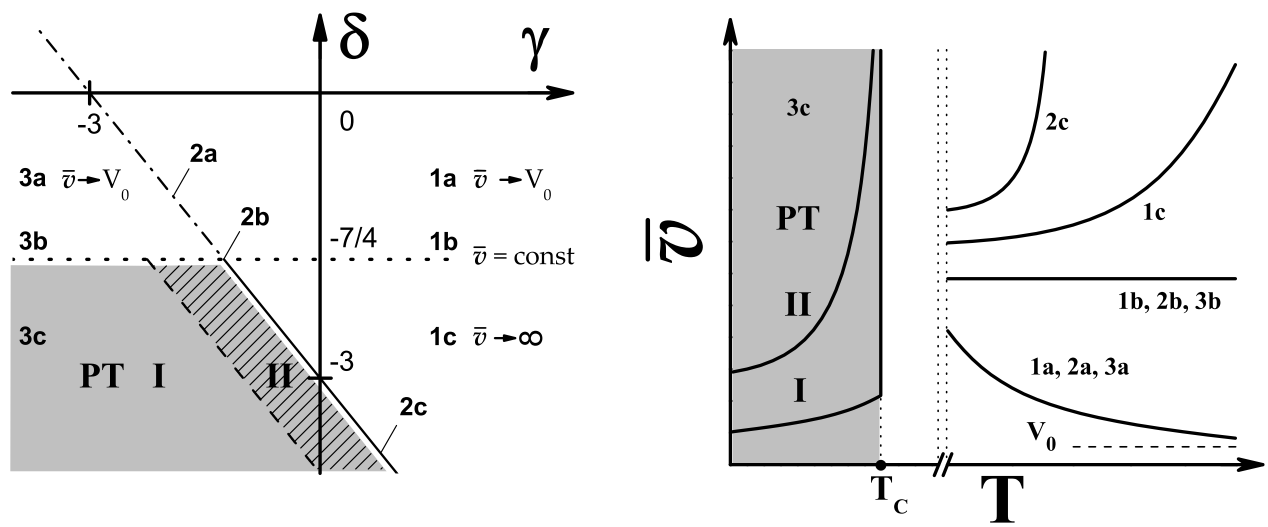

FIG. 1: *Left:* The $\gamma - \delta$ phase diagram for the gas of quark-gluon bags. *Right:* The schematic view of temperature dependence of the average volume of the bag in different regions of $\gamma - \delta$ phase diagram. See text for details.

Most interest in the previous studies [3, 4, 5, 6, 7, 8] was devoted to PTs. Different order PTs are discussed in details in Ref. [5]. We concentrate here on the situations when PTs are

absent. Eq. (6) for $s_H(T)$ can be written as follows:

$$s_H \;=\; u(T)\,\int_{V_0}^{\infty} dv\; v^{a-1}\;\exp\left(-v\;\Delta s\right)\;\propto\; T^{10+4\delta}\;\left(\Delta s\right)^{-a}\;\Gamma(a, V_0 \Delta s)\;, \tag{11}$$

where $a \equiv \gamma + \delta + 3$, $\Delta s(T) \equiv s_H - s_Q$, and $\Gamma(a, b)$ is the incomplete Gamma-function. The function $f(T, s)$ has the singular point $s = s_Q$, and $f(T, s_Q)$ can be either finite or infinite depending on the value of $\gamma + \delta$:

$$1)\;\; \gamma+\delta > -3\;, \qquad 2)\;\; \gamma+\delta = -3\;, \qquad 3)\;\; \gamma+\delta < -3\;. \tag{12}$$

The phase transition never exists for $\gamma + \delta \geq -3$ because of $f(T, s_Q) = \infty$, and in this case the solution $s_H(T)$ of Eq. (11) always corresponds to $s_H > s_Q$, see Fig. 2 *left*. For $\gamma + \delta < -3$ the existence of a PT depends on the value of parameter δ. There are three distinct cases:

$$a)\;\; \delta > -7/4\;, \qquad b)\;\; \delta = -7/4\;, \qquad c)\;\; \delta < -7/4\;. \tag{13}$$

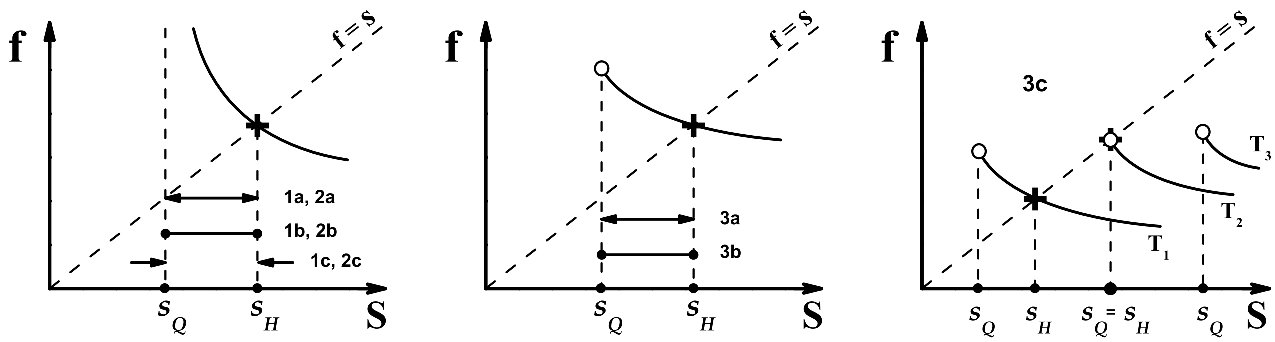

FIG. 2: The solid lines present the dependence of f on s at fixed temperatures. The pole singularity s_H and singularity s_Q are denoted by crosses and circles, respectively. *Left:* The cases 1 and 2 in Eq. (12) correspond to $f(T, s_Q) = \infty$. *Middle:* The cases 3a and 3b in Eqs. (12,13) are shown. They correspond to $f(T, s_Q) > s_Q$ at all T. *Right:* The case 3c with $T_1 < T_2 = T_C < T_3$. It leads to the "collision" of two singularities $s_H = s_Q$ at the PT temperature T_C.

The PTs take place in the system of quark-gluon bags in the only case 3c, i.e. if $\gamma + \delta < -3$ and $\delta < -7/4$. This is the lower left corner of the $\gamma - \delta$ phase diagram marked as 3c and shown by grey color in Fig. 1 *left*. The region with 1st and 2nd or higher order PTs are marked as I and II correspondingly.

The average volume of the quark-gluon bag can be calculated as:

$$\overline{v}(T) \;=\; \frac{\int dvdm\; v\; \rho(m,v)\;\phi(T,m)\;\exp(-s^{*}v)}{\int dvdm\; \rho(m,v)\;\phi(T,m)\;\exp(-s^{*}v)} \;\cong\; \frac{1}{\Delta s(T)}\,\frac{\Gamma(a+1, V_0\,\Delta s(T))}{\Gamma(a, V_0\,\Delta s(T))}\;. \tag{14}$$

It can be proven that $s_H \sim T^3$ for $T \to \infty$. Using the asymptotic expansion for incomplete Γ-function (see Appendix) one obtains from Eqs. (11), (14) the behavior of average volume $\overline{v}(T)$ of the quark-gluon bag at large temperature T:

$$1a,\ 2a,\ 3a\ :\quad \Delta s(T)\ \sim\ \ln T\ \to \infty\ ,\qquad \overline{v}(T) \to V_0\ , \tag{15}$$

$$1b,\ 2b,\ 3b\ :\quad \Delta s(T)\ \cong\ const\ >\ 0\qquad \overline{v}(T) \to const\ , \tag{16}$$

$$1c\ :\quad \Delta s(T)\ \sim\ T^{(7+4\delta)/a}\ \to\ 0\ ,\qquad \overline{v}(T) \sim T^{-(7+4\delta)/a}\ \to\ \infty\ , \tag{17}$$

$$2c\ :\quad \Delta s(T)\ \sim\ \exp\left(-\ T^{-7-4\delta}\right)\ \to\ 0\ ,\qquad \overline{v}(T)\ \sim\ \exp\left(T^{-7-4\delta}\right)\ \to\ \infty\ . \tag{18}$$

The results (15-18) for $\overline{v}(T)$ are schematically presented in Fig. 1 *right*. Note that if the phase transition takes place at $T = T_C$ (case 3c) the $\overline{v}(T)$ becomes infinite at $T > T_C$ in the thermodynamic limit. This is also shown in Fig. 1 *right*.

IV. SUMMARY

In this paper we have studied the gas of quark-gluon bags at high temperature T. The mass-volume spectrum function $\rho(m, v)$ for the quark-gluon bags is taken in the form (8). The behavior of the system depends crucially on the values of the γ and δ parameters in Eq. (8). The special regions of the $\gamma - \delta$ phase diagram defined by Eqs. (12,13) lead to different behavior of the gas of the quark-gluon bags. The pressure p and energy density ε for different values of γ and δ have the same asymptotic behavior at high temperature, $p \cong \sigma_Q T^4/3$ and $\varepsilon \cong \sigma_Q T^4$. This corresponds to the equation of state of non-interacting quarks and gluons inside the bags, i.e. *ideal* quark-gluon plasma (QGP). However, the average volume of the bag $\overline{v}(T)$ (15-18) and average mass $\overline{m}(T) \cong \overline{v}(\sigma_Q T^4 + B)$ have rather different behavior in different regions of the $\gamma - \delta$ phase diagram. If the system of quark-gluon bags has no phase transition the $\overline{v}(T)$ remains finite at high temperature. Such a *cluster* QGP can be rather different from the *ideal* QGP despite of the similar to that equation of state. The kinetic properties of the *cluster* QGP, e.g. the shear and bulk viscosity, may deviate strongly from those in the quark-gluon gas.

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APPENDIX A

The incomplete gamma function $\Gamma(a,x)$ (see, e.g., Ref. [12]) has the asymptotic expansions at $x \to \infty$,

$$\Gamma(a, x \to \infty) \;=\; x^{a-1} e^{-x} \left(1 \;+\; \frac{a-1}{x} \;+\; O(x^{-2}) \right) \;, \tag{A1}$$

and $x \to 0$,

$$\begin{aligned} \Gamma(a, x \to 0) \;&=\; \Gamma(a) \;-\; \frac{x^a}{a} \left(1 \;+\; O(x)\right) \;, && a \neq -n \;, \\ &=\; \frac{(-1)^n}{n!} \left(\psi(n+1) \;-\; \ln x\right) \;+\; \frac{x^{-n}}{n} \left(1 \;+\; O(x)\right) \;, && a = -n \;, \end{aligned} \tag{A2}$$

where $\psi(z)$ is the logarithmic derivative of the gamma function, $\psi(z) = \Gamma'(z)/\Gamma(z)$. Eq. (A2) gives for $a = 0,\ -1,\ -2$:

$$\begin{aligned} \Gamma(a, x \to 0) \;&=\; -\ln x \;+\; O(1) \;, && a = 0 \\ &=\; \ln x \;+\; \frac{1 + O(x)}{x} \;, && a = -1 \;, \\ &=\; \frac{1 + O(x)}{2x^2} \;, && a = -2 \;. \end{aligned} \tag{A3}$$

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